Analysis of 3D pendulum sliding along a rope

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EXTENDED ABSTRACT

1 Introduction

The article discusses a dynamic engineering problem in which a mass attached to a pendulum slides along an elastic rope. This type of problem describes many practical applications such as weight manipulation, construction site transportation, cableway, etc. Usually, the center of the mass is almost always away from the rope axis, which is modeled with a pendulum attached to the rope. A similar problem was analyzed in [1] and this is the 3D extension of this work.

The pendulum mass and the rope are coupled in a model described by a system of algebraic differential equations (DAE). The initial conditions for the given system of DAE are formulated and solved separately. From the literature, finite elements are the preferred method for solving structural problems, but here the moving mass is a part of the structure, which requires the development of special finite elements [2]. An analytical formulation of the problem is not found in the literature. Note that sliding along the wire should be distinguished from sliding of the wire itself [3].

The developed model is general in the sense of free choice of support location, elastic wire properties, pendulum length, and inclusion of braking forces (or coefficient of friction). Further generalization would include multiple successive masses [4]. The model is verified and illustrated with examples.

The novelty of this approach consists in the combination of the two independent formulations: i) sliding of a mass along an extensible rope and ii) motion of a 3D pendulum in the translational coordinate system. Moreover, this analytical approach could be used to verify different discrete formulations of the problem.

2 Sliding pendulum model

The rope imposes nonlinear constraints onto dynamic equations of mass movement and mass sliding along the rope and mass swinging in the pendulum are combined into a system of differential algebraic equations.

The sliding mass model assumes a straight rope, i.e. the dead weight of the rope is neglected and the rope sag is rather small. The description of the dynamic equilibrium of the mass yields three second order differential equations relating the mass accelerations in three coordinate directions to the kinematic constraints of the rope.

The pendulum model describes the 3D oscillation of a mass on an inextensible wire. The model is formulated in a local {x,y,z} coordinate system, where '*z*' is not an independent coordinate because we have $z^2 = L_p^2 - x^2 - y^2$ due to the inextensible pendulum wire of length 'L_p'. The dynamic equilibrium is described with conservation of momentum around two independent axes of rotation (along and perpendicular to the wire): ϑ and ψ that leads to two second order differential equations [5].

The combined model connects the mass sliding along an extensible wire to a 3D pendulum model. The connecting equations are given in equation (1):

$$\begin{Bmatrix} \ddot{a} \\ \ddot{Y} \\ \ddot{F} \end{Bmatrix} = \begin{Bmatrix} DE_1 \\ DE_2 \\ DE_3 \end{Bmatrix} - \begin{Bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{Bmatrix}. \tag{1}$$

where DE_i are differential equations describing changes in position along the wire. {x,y,z} are pendulum coordinates that have to be connected with the coordinates of the point on the wire, equation (2):

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = L_p \begin{Bmatrix} sin(\psi(t)) \\ cos(\psi(t)) sin(\upsilon(t)) \\ cos(\psi(t)) sin(\upsilon(t)) \end{Bmatrix}. \tag{2}$$

After introducing the necessary transformations, our model is described by a system of 10 normalized (first order) nonlinear differential equations with appropriate boundary conditions. The variables in the equations are listed in Table 1.

The initial conditions determine the dependent parameters, given the initial position of the load (the pendulum). It is determined by two nonlinear equations with unknown total sag F and wire tension force T. One equation describes the total length of the rope (including elastic elongation) and the other describes the force balance at the load position. It is possible to combine the two equations into one by inserting the length constraint into the force balance.

Table 1: Unknowns in the system of equations

Variable	*meaning*	*Variable*	*meaning*
a	position on the rope	u	longitudinal velocity of rope at ‘a’
F	displacement of the rope at 'a'	w	vertical velocity of rope at ‘a’
Y	lateral displacement of the rope at 'a'	v	lateral velocity of rope at ‘a’
ϑ	pendulum angle along to the rope	p	velocity (speed of change) of ϑ
ψ	pendulum angle perpendicular to the rope	q	velocity (speed of change) of ψ

3 Example

An example was calculated using Wolfram Mathematica [6] and the first 42 seconds of the simulation are shown in Fig.1. The trajectories of the pendulum suspension point and the pendulum mass on the 600 m long zip-line are shown.

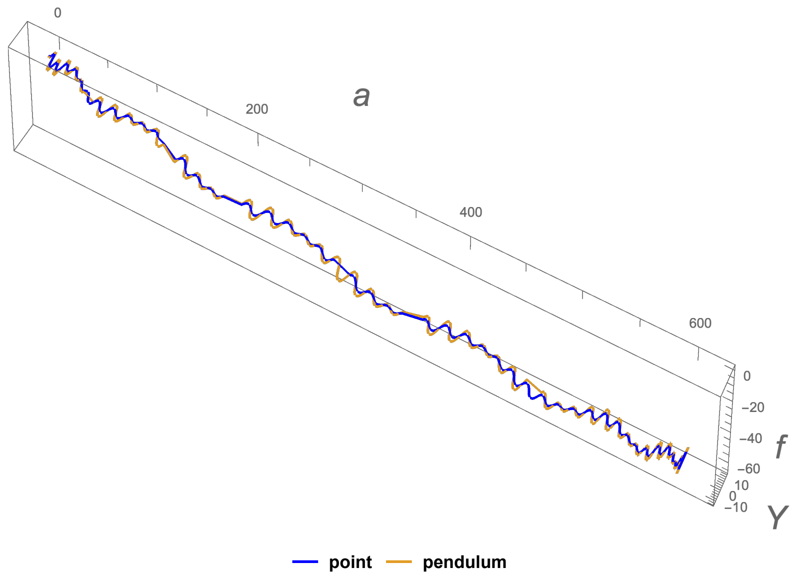

Figure 1: Some of the solution functions

Fig.1 shows that the mass point hits the end of the rope and bounces back. In the real situation, the speed would be reduced by braking and the end of the rope would be reached smoothly.

Acknowledgments

This work was supported by project KK.01.1.1.04.0056 "Structure integrity in energy and transportation” and project uniri-tehnic-18-108-1245, for which we gratefully acknowledge.

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